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1. The Scope of Benoît Mandelbrot's Work and its Influence

> Why is geometry often described as cold and dry? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.
>
> More generally, I claim that many patterns of Nature are so irregular and fragmented, that, compared with *Euclid*—a term used in this work to denote all of standard geometry—Nature exhibits not simply a higher degree but an altogether different level of complexity. The number of distinct scales of length of natural patterns is for all practical purposes infinite.
>
> The existence of these patterns challenges us to study those forms that Euclid leaves aside as being formless, to investigate the morphology of the amorphous. Mathematicians have disdained this challenge, however, and have increasingly chosen to flee from nature by devising theories unrelated to anything we can see or feel.

These paragraphs, by which Benoît Mandelbrot began his book, *The Fractal Geometry of Nature* (1982), have become some of the most widely quoted lines about mathematics and nature, and Mandelbrot's unusually beautiful book has become one of the classics of 20th century mathematics. Mandelbrot was one of the most famous mathematicians of the second half of the twentieth century. It is difficult to think of any other twentieth century mathematician who published on such a wide range of subjects, both for specialists and also for the general educated public. From 1951 until near the end of his life Mandelbrot published not only in mathematics, but in diverse fields between which he found relationships and connections, including information theory, linguistics, mathematical economics, physics, chaos theory, and even cosmology. The common thread through all of his work was describing mathematically what Mandelbrot characterized as "roughness"-- the way that objects in nature, such as mountains, or the branching of plants or trees, or blood vessels in the human body, or even the human pulse, reflect consistently irregular or rough patterns rather than smoothness or straight lines with which traditional Euclidean geometry is concerned. Mandelbrot's geometry described objects that are equally rough at all scales. His ways of describing roughness also applied to the description of errors in information transmission, and to the irregular behavior of prices and the stock market. Mandelbrot showed how complex patterns were repeated over and over in the widest variety of structures both abstract, such as the Mandelbrot set, and in living things. Calculating the enormous numbers of iterations necessary to visualize these complex patterns required electronic computing, the use of which Mandelbrot pioneered for the development of geometry.

Mandelbrot is, of course, best known as the founder of fractal geometry. In 1976 he coined the term fractal, which he derived from the Latin adjective, *fractus*, meaning "to break" or "to create irregular fragments," for self-similar patterns, where self-similar means they are "the same from near as from far." Fractals may be exactly the same at every scale, or they may be nearly the same at different scales. The concept of fractal includes the idea of a detailed pattern repeating itself. Besides geometric patterns, for which fractals are mostly widely known, they can describe processes in time, in structures and in sound. In nature these patterns are typically so complex that they could not be analyzed in detail without electronic computers. However, Mandelbrot's papers were often theoretical and mathematical in nature, independent of computer programs.

Mandelbrot led a fascinating intellectual life, and spoke and often wrote about his life and work with great personal charm. In an interview with Anthony Barcellos published in Albers and Alexanderson's *Mathematical People: Profiles and Interviews* (1986) Mandelbrot had this to say about some of his early work:

> My wild gamble started paying off during 1961-62. By then, there was no question in my mind that I had identified a new phenomenon present in many aspects of nature, but all the examples were peripheral in their fields, and the phenomenon itself eluded definition. To denote it, the usual term is the Greek 'chaos,' but I was using the weaker-sounding Latin term 'erratic behavior' at the time. The better word 'chaos' came later from others, but I was the first to focus on the underlying notion, and to specialize in studying the erratic-chaotic. Many years were to go by before I formulated fractal geometry, and became able to say that I had long been concerned with the fractal aspects of nature, with seeking them out and with building theories around them.
>
> But let us go back to the year 1961. Starting in that year, I established that the new phenomenon was central to economics. Next, I established that it was central to vital parts of physical science, and moreover that it involved the concrete interpretation of the great counterexamples of analysis, and finally, I found that it had a very important visual aspect. I was back to geometry after years of analytic wilderness! A later turning point came when I returned to questions of interest to those in the mainstream of mathematics.

In 1963 Mandelbrot found recurring patterns at every scale in data on cotton prices. This he published in "The Variation of Certain Speculative prices," *The Journal of Business* 36 (1963) 394-419. Around the same time in studying patterns in information transmission over telephone lines he concluded that on any scale the proportion of noise-containing periods to error-free periods was a constant – thus errors were inevitable and must be planned for by incorporating redundancy. This he published in J. M. Berger and Benoit Mandelbrot, "A New Model for the Clustering of Errors on Telephone Circuits," *IBM Journal of Research and Development* 7 (1963) 224-36.

In 1965 Mandelbrot published "Information Theory and Psycholinguistics." This paper, issued as a chapter in Wolman and Nagel's *Scientific psychology* (1965) generalized a power-law distribution on ranked data named after linguist George Kingsley Zipf, and formulated the Zipf-Mandelbrot law, with applications far outside beyond information theory or linguistics, such as ecological field studies in which the relative abundance distribution (i.e. the graph of the number of species observed as a function of their abundance) is often found to conform to a Zipf–Mandelbrot law, and within music, in which many metrics of measuring "pleasing" music conform to Zipf-Mandelbrot distributions. The law is the basis of many approaches to electronic data compression.

In papers published between 1965 and 1973 concerning hydrology and the appearance of periodicity in discharges and floods of the Nile river Mandelbrot described both the "Noah effect," in which sudden discontinuous changes can occur, such as floods, and the "Joseph effect," in which persistence of a value can occur for a while, such as famines, yet suddenly change afterwards. This challenged the idea that changes in price were normally distributed. (Mandelbrot cited his specific papers on this topic in *The Fractal Geometry of Nature* [1982] 248, but see especially Mandelbrot & Wallis, "Noah, Joseph and Operational Hydrology," *Water Resources Research* 4 [1968] 909-18). This work had implications for climatology, for agriculture, for the design of dams, and on economic issues.

In 1974 Mandelbrot offered a new explanation of Olbers' paradox (the "dark night sky"riddle) demonstrating the consequences of fractal theory as a sufficient, but not necessary, resolution of the paradox. The night sky can be viewed as one of the most beautiful fractals in nature. Wherever you look there is a star, and between any two stars there are always other stars. In 1826 Wilhelm Olbers argued that in a large enough universe the sky ought to be uniformly bright. Because luminosity decreased with the quare of the distance, and so does apparent size, the total amount of light coming from any direction ought to be the same. Mandelbrot postulated that if the stars in the universe were fractally distributed it would not be necessary to rely on the Big Bang theory to explain the paradox. His model did not rule out a Big Bang, but allowed for a dark sky even if the Big Bang had not occurred.

Mandelbrot's theories of self-similarity first began to be more widely known in 1967 with the publication of the large circulation journal *Science* of his paper "How Long is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension," *Science*. New Series. Vol. 56., No. 3775. May 5, 1967 636-38. In this paper Mandelbrot showed that a coastline's length varies with the scale of the measuring instrument, resembles itself at all scales, and is infinite in length for an infinitesimally small measuring device. Arguing that a ball of twine appears to be a point when viewed from far away (0-dimensional), a ball when viewed from fairly near (3-dimensional), or a curved strand (1-dimensional), he argued that the dimensions of an object are relative to the observer and may be fractional. An object whose irregularity is constant over different scales ("self-similarity") is a fractal.

Today Mandelbrot's paper on the coast of Britain is famous in the history of mathematics. Yet when it was published it was mainly misunderstood. Years later Mandelbrot had this to say about the origin of the paper:

> By the mid-1960s my record of publications was substantial but presented a serious flaw. Those publications' topics ranged all too widely and were perceived as an aimless juxtaposition of studies of noise, turbulence, galaxy clustering, prices and river discharges. Few persons realized that, to the contrary, I did not deserve to be criticized for immature aimlessness but for increasingly acute single-mindedness. As early as 1956. . . .then increasingly and more seriously in my works on finance and on noise, I had somehow latched on the process of renormalization and found it useful in very diverse contexts. Unfortunately, the nature and worth of that concept was not appreciated until much later when it was rediscovered quite independently in the statistical physics of critical phenomena that arose in 1972.
>
> More specifically, nearly all my works were linked by the ubiquity of 'power-law' relations, each endowed with an important exponent. Superficially those exponents seemed both formal and mutually unrelated. But in fact I knew how to interpret them geometrically as 'the' fractal dimension of suitable sets. Furthermore, this interpretation gave to my work a profound unity that promised further growth. But I soon found out that mention of a fractal dimension in a paper or a talk led all referees and editors to their pencils, and some audiences to audible signs of disapproval. Practitioners accused me of hiding behind formulas that were purposefully incomprehensible. Few mathematicians knew any of the flavors of fractal dimension; if asked, they were worse than useless in explain this notion to those I was trying to convert. . . .
>
> Fortunately, I stumbled one day upon Richardson's empirical data on coastline lengths, and recognized instantly that a study of coastlines might lend itself to a 'Trojan horse' manoeuver. Indeed, everyone has knowledge of geography, but no one I knew professionally had a vested professional interest in facts and theories concerning coastlines and relief. The manoeuver succeeded. Everyone was wonderfully objective and receptive to the seemingly wild idea contained this paper, and as a result, became more receptive to the use of fractal dimension in fields that really matter to me.
> (http://users.math.yale.edu/~bbm3/web_pdfs/howLongIsTheCoastOfBritain.pdf)

In his 1975 French book, *Les objets fractals: Forme, hasard et dimension* Mandelbrot first coined the term "fractal." He revised, expanded and translated these ideas in his 1977 English language book, *Fractals: Form, Chance and Dimension*, and expanded them further in his 1982 book, for which is he is most famous, *The Fractal Geometry of Nature*. Though I have not been able to source the quotations, in 1999 *American Scientist* magazine stated that Mandelbrot's 1977 book was "one of the hundred most influential science books" of the 20th century, and that Mandelbrot's three books published from 1975 to 1982, taken together, comprise "one of the ten most influential scientific essays of the 20th century." The impact of these books on the scientific community, and on the educated public, was significantly enhanced by mathematically accurate computer-drawn illustrations created by programmers working with Mandelbrot, primarily at IBM Research. Images for the 1977 and 1982 books were mainly by physicist Richard F. Voss in association with Mandelbrot. The early graphics were low-resolution black and white; later drawings were higher resolution and in color as computer graphic technology evolved between 1975 and 1982. Regarding Mandelbrot's 1977 book Freeman Dyson wrote in 1978:

> Fractal is a word invented by Mandelbrot to bring together under one heading a large class of objects that have [played] . . .an historical role . . .in the development of pure mathematics. A great revolution of ideas separates the classical mathematics of the 19th century from the modern mathematics of the 20th. Classical mathematics had its roots in the regular geometric structures of Euclid and the continuously evolving dynamics of Newton. Modern mathematics began with Cantor's set theory and Peano's space-filling curve. Historically, the revolution was forced by the discovery of mathematical structures that did not fit the patterns of Euclid and Newton. These new structures were regarded . . . as 'pathological,' . . .as a 'gallery of monsters,' kin to the cubist paintings and atonal music that were upsetting established standards of taste in the arts at about the same time. The mathematicians who created the monsters regarded them as important in showing that the world of pure mathematics contains a richness of possibilities going far beyond the simple structures they saw in Nature. Twentieth-century mathematics flowered in the belief that it had transcended completely the limitations imposed by its natural origins.
>
> Now, as Mandelbrot points out. . . Nature has played a joke on the mathematicians. The 19th century mathematicians may have been lacking in imagination, but Nature was not. The same pathological structures that the mathematicians invented to break loose from the 19th century naturalism turn out to be inherent in familiar objects all around us (Dyson, "Characterizing Irregularity," *Science* 200, 4332 (1978) 677-78).

In his Foreward to Peitgen's, *The Science of Fractal Images* (1998) Mandelbrot stated (p.8) that as early as 1972 he worked with Hirsch Lewitan on film clip "on the creation of fractal galaxy clusters, using the *Seeded Universe* method. Then, in 1975, with Sig Handelman, we added a clip in which the landscape to be later used on Plate 271 of the *Fractal Geometry of Nature* emerged slowly from the deep, then rotated majestically (or at least very slowly), and finally slipped back under water." These very short pioneering computer graphic films had to be made by recording computer monitor images on film.

One of the most obvious aspects of computer graphics created using fractals was the way they can create realistic wrinkles that look like real natural objects. Mountain climbers are often misled by lack of distinguishing features which can make a peak appear to be just over the next rise, only to find a vast chasm between them and the top. This is because any part of a mountain resembles the whole. In the wider film animation industry the influence of Mandelbrot's 1977 book on fractals on the development of computer graphics was dramatic. Prior to fractals computer graphics engineers had struggled to render natural images on computers, and it had been impossible to render enough images to create natural looking films using computer graphics. Graphics in films had to be done by hand, frame by frame. After reading Mandelbrot's *Fractals: Form, Chance and Dimension* during 1979 and 1980 Loren Carpenter created a two-minute color film called *Vol Libri* to showcase his software for rendering realistic mountains and landscapes using fractal geometry at a SIGGRAPH conference in 1980. This was the first application of fractals in a computer-animated film. Production of each frame in the two-minute film required 20—40 minutes of computing time on a VAX-11/780 computer. As a result of this film Carpenter was hired by George Lucas's Industrial Light and Magic where he created an entire fractally-landscaped planet in the first completely computer-generated cinematic image sequence in a feature film, *Star Trek II: The Wrath of Khan* issued in 1982. While most viewers of this film probably did not appreciate the pioneering application of fractal geometry involved in that cinematic sequence, the technology became more and more widely applied in computer graphic animation now used in all kinds of movies; Carpenter is currently co-founder and chief scientist of Pixar Animation Studios.

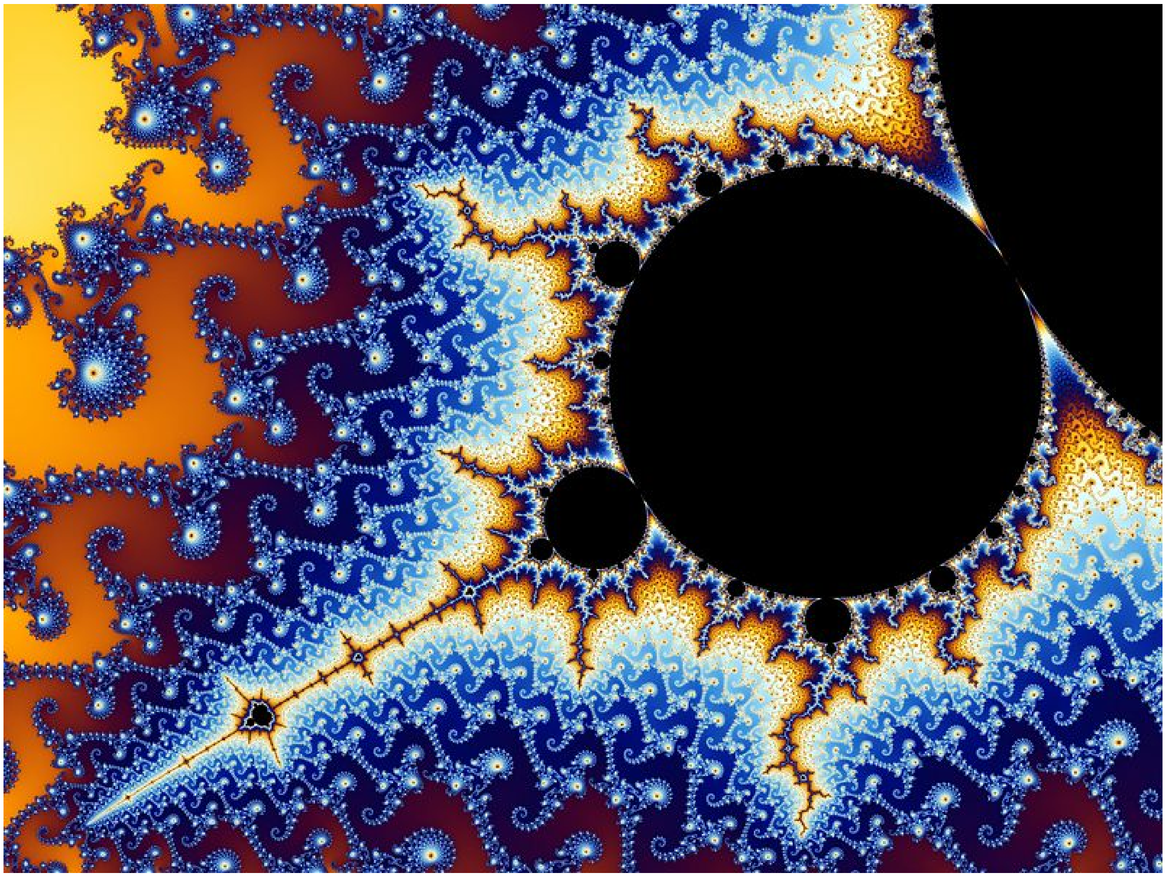
Regarding his contribution to the development of computer graphics Mandelbrot had this to say in an interview published on the Internet in the Web of Stories website (Mandelbrot, Web of Stories, 8 "Drawing: the ability to think in pictures and its continued influence":

> **. . . But I didn't know that I could draw, but in that preparation for the École Normale and the École Polytechnique, at the time drawing was just part of the programme. It sounded ridiculous because there was no need for it, but again it was tradition. Once upon a time engineers had to be able to draw the state of something happening for their bosses, or, if they're coming to inspect a bridge being built, to draw what was happening. Therefore drawing was an important part of the game, and to train that skill in a kind that could be also subjected to exams, we had - well, the Venus de Milo, the Victory of Samothrace, the head of Voltaire, etc., etc., all kinds of classical and French sculptures to imitate. And I found that I could do it very accurately. It was rather soulless but extremely accurate and extremely careful depiction. All that was part of this complex of skills which again,**

amazingly enough, I did not know about before, namely the ability to draw, to see things in accurate detail, to see differences between my drawing and the model very accurately, and to think in terms of pictures. I might say that this had been my skill throughout, that in all the very complicated ups and downs of my life the ability to think globally- certain configurations has been predominant. The ability, the willingness, to ask myself questions about what shapes things are, because then I could think about them, had been predominant. In my work in pure mathematics most of it - the parts most exciting for mathematicians - has been parts in which I was asking questions which nobody else had asked before, because nobody else had actually looked at certain structures. Therefore, as I will tell, the advent of the computer, not as a computer but as a drawing machine, was for me a major event in my life. That's why I was motivated to participate in the birth of computer graphics, because for me computer graphics was a way of extending my hand, extending it and being able to draw things which my hand by itself, and the hands of nobody else before, would not have been able to represent.
http://www.webofstories.com/play/10535?o=FHP

The most famous fractal is that known as the "Mandelbrot set." It is also the most complex object in mathematics, and one of the most beautiful mathematical objects known. Theoretically it can be magnified to a size greater than the universe, and it keeps changing visually as it is magnified or reduced, but since it is only a mathematical object you cannot touch it. Computer graphic images of the Mandelbrot set (generated from the equation $X_n +_1 = X_n^2+C$) display an elaborate boundary that reveals progressively ever-finer recursive detail at increasing magnifications. Describing the computer graphic visualizations of the Mandelbrot set, the "style" of this repeating detail depends on the region of the set being examined. The set's boundary also incorporates smaller versions of the main shape, so the fractal property of self-similarity applies to the entire set, and not just to its parts. Because of its haunting beauty when displayed by computer graphics, the Mandelbrot set has become the most famous object in modern mathematics--an inspiration for artists, and a source of wonder for schoolchildren. Mandelbrot first saw the visual record of the Mandelbrot set in a low resolution plotter image generated in March 1980 while he was Visiting Professor of Mathematics at Harvard. "The basement of the Science Center housed its first Vax computer (brand new and not yet 'broken in'); to view the pictures, we used a Tektronix cathode ray tube (worn out and very faint), and our hard copies were printed on a Versatec device no one knew how to set up properly" (Mandelbrot, *Frontiers of Chaos*, 12). Through progressively more detailed images in April and May, and finally a higher resolution image produced in June 1980, the enormous visual potential of the set began to manifest itself. Remarkably Mandelbrot first visualized the set on lower quality equipment available at Harvard rather than the state of the art equipment he had available at IBM Research, Yorktown Heights.

An image of the Mandelbrot set generated much later:



Mandelbrot applied his fractal geometry and mathematics to a wide variety of fields, and published a very large number of papers. His papers in different fields were collected later in his life in a series of 4 volumes which included bibliographical listings of his papers in those particular fields. I have not found a comprehensive listing of all of his publications, but if all his scientific papers, magazine articles and books were added up they would amount to several hundred. The published volumes of Mandelbrot's selected works are:

- *Fractals and Scaling in Finance: Discontinuity, Concentration, Risk (1997)*
- *Multifractals and I/f Noise: Wild Self-Affinity in Physics (1999).*
- *Gaussian Self-Affinity and Fractals* (2001),
- *Fractals and Chaos. The Mandelbrot Set and Beyond* (2004)

A remarkably large number of scientists have applied fractals to diverse fields of science including medicine, neuroscience, pathology, enzymology, geology, oil exploration, geography, archaeology, soil mechanics, seismology, and even search and rescue. In technology they have been applied in the design of fractal antennas, in digital imaging, in signal and image compression, in computer and video game design, in computer graphics, in networks, and in diagnostic imaging.

Fractals can be found in natural phenomena including clouds, coastlines, ocean waves, earthquakes, river networks, fault lines, mountain ranges, craters, lightning bolts, various vegetables, especially cauliflower and broccoli, animal coloration patterns, heart rates, snowflakes, crystals, blood vessels and pulmonary vessels, and DNA.

In the arts, fractals have been applied in electronic music. Perhaps more remarkably, even though the paintings of American artist Jackson Pollock appear to be composed of chaotic dripping and splattering, computer analysis has found fractal patterns in Pollock's work.

Mandelbrot first promoted the scientific and aesthetic/visual aspect of fractals in his three books published between 1975 and 1982, but for him the mathematics was always more significant than its graphical representation by computer. In the last of these books (1982) he wrote (p. C16):

> Computer graphics played a central role in the acceptance of fractal geometry, but a peripheral role in its genesis. That is, granted the fascination that fractals now hold for the computer practitioners, one is tempted to credit the emergence of the new geometry to the availability of this new tool. Actually, I formulated the theory of fractals when computer graphics was in its infancy. However, I let its development be biased toward topics that lend themselves to intuition-building illustrations.

Probably the full aesthetic impact of fractals was first felt by large numbers of the general public when the exhibition, *Frontiers of Chaos: Computer Graphics Face Complex Dynamics*, was held in Germany and the United States in New York and San Francisco in 1985. This exhibition, with its powerful dramatic color graphics, was viewed by no less than 140,000 people. The following year the editors of the beautiful bilingual exhibition catalogue, H. –O Peitgen and P. H. Richter, from the University of Bremen, issued an even more spectacular book, *The Beauty of Fractals. Images of Complex Dynamical Systems* (1986). At the beginning of his chapter entitled "Refractions of Science into Art"a contributor to the exhibition catalogue and book, the computer graphics expert Herbert W. Franke, had this to say about the place of fractals in computer graphics and visual art generally:

> Art critics in the centuries to come will, I expect, look back on our age and come to conclusions quite different than our own experts. Most likely the painters and sculptors esteemed today will nearly have been forgotten, and instead the appearance of electronic media will be hailed as the most significant turn in the history of art. The debut of those first halting and immature attempts to achieve that ancient goal, namely the pictorial expression and representation our world, but with a new media, will finally be given due recognition.

It will be pointed out that back then (now!) it became possible for the first time to create three dimensional pictures of imaginary landscapes and other scenes with photographic precision, and with these pictures not just to capture an instant in time but to include the reality of change and movement. Perhaps this is the most important aspect of the new turn; the time dimension has been unlocked for pictures, and planar or three dimensional scenes in perspective, even from points of view not accessible to the human eye or camera, can be arranged freely. . .

During his adjunct professorship at Yale beginning in 1987 Mandelbrot collaborated with programmers and artists in advancing methods of producing more realistic fractal images. In a notable paper he authored with F.K. Musgrave, "The art of fractal landscapes," *IBM Journal of Research and Development* 35 (1991) No. 4, 535-40, with color fractal images published on the upper and lower covers of the issue, he worked to "improve models of natural phenomena for computer graphics and to set a new standard for both realism and aesthetic quality in computer-synthesized landscape images." (p. 535).

The images on the covers and interior color fold-out of the offprint of this paper published in 1991 are remarkably natural looking, even painterly, compared to those generated ten years earlier for the *Fractal Geometry of Nature* (1982).

Because of Mandelbrot's work, the amount of scientific research on fractals exploded in so many fields of science and art; today that it is far too extensive to summarize. As an example of the number of books in the English language published on the subject, on June 9, 2012 Amazon.com listed 3,227 titles for sale concerning fractals, virtually all of which are building on Mandelbrot's work.

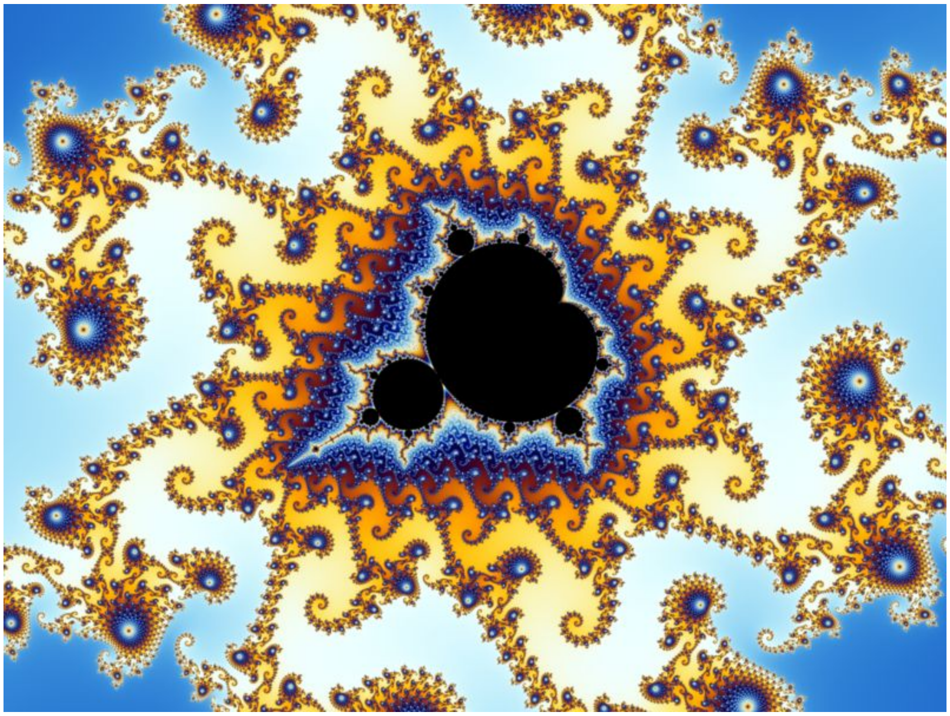
2. A Researcher at IBM Who Never Touched a Computer

Considering that Mandelbrot spent thirty-five years on the staff of IBM Research, it is worth noting that he never personally touched a computer. Certainly he understood how computers operated, and used them, with the aid of assistants, extensively in his work. The archive contains perhaps 200 rolls of computer print-outs, showing algorithms run on computers used in his research; however, there is no evidence in the archive that Mandelbrot ever touched a keyboard of any kind, including a typewriter keyboard, instead writing everything by long hand. It is possible that Mandelbrot was the last great mathematician to do all his composition in handwriting. His style was to write by hand, to have each draft typed by a secretary, and to make manuscript corrections on successive drafts which would then be retyped by the secretary. One of the most remarkable aspects of the archive is that so many, if not all drafts, of each paper were retained, illustrating his process, and Mandelbrot published hundreds of papers. Retention of a very extensive paper record was also characteristic of Mandelbrot's organization of his correspondence with others. Even after email became widely used his secretary would print out all incoming email. Mandelbrot would respond on paper, which would be keyed in by his secretary, retaining a print-out of outgoing correspondence. Because of his writing style, this archive contains perhaps 20,000 or more pages of writing by Mandelbrot with the text all or in part in his handwriting. It is probably that last archive of a famous late 20th century mathematician that is preserved entirely on paper.

3. A Science Archive Significant for "Book History"

That Mandelbrot retained most drafts of his papers and books, from their initial conception through publication, makes his archive an excellent, if unusual source for the book history of the period from about 1975 through about 2010. Especially valuable are the records of development of his books on fractals which go from the earliest manuscript drafts all the way through the pasted-up boards for photo-offset printing, and include all the original art for these extremely visual books. The collection is also significant for its very large collection of computer-generated images done under Mandelbrot's direction but without his hands-on participation at the computers.

Another image of the Mandelbrot set:



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The Colours of Infinity. The Beauty and Power of Fractals. Introduced by Arthur C. Clarke [1994?]. Includes a DVD narrated by Clarke, which runs on British players only, it seems. Regardless of this technical problem, this is the most beautiful book on fractals that I have found. When I prepared this appraisal a low resolution version of *The Colours of Infinity DVD* was available on the internet at http://www.misterx.ca/Mandelbrot_Set---Thumb_Print_of_God.html. It was also available on YouTube.

The Fractal Revolution with Benoit Mandelbrot. Reprint from The Institute for Science, Engineering and Public Policy, n.p., n.d.

www.webofstories.com/story/search?q=mandelbrot. This is the most extensive video interview broken into 109 segments of 3-4 minutes each.

Wikipedia articles on Benoit Mandelbrot, Chaos Theory, Fractals, Zipf-Mandelbrot Law, Mandelbrot Set